Generalised cosmology of codimension-two braneworlds

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Abstract

It has recently been argued that codimension-two braneworlds offer a promising line of attack on the cosmological constant problem, since in such models the Hubble rate is not directly related to the brane tension. We point out challenges to building more general models where the brane content is not restricted to pure tension. In order to address these challenges, we construct a thick brane model which we linearize around a well known static solution. We show that the model's cosmology does reduce to standard FRW behaviour, but find no hint of a self-tuning mechanism which might help solve the cosmological constant problem whithin the context of non-supersymmetric Einstein gravity.

1 Introduction

It is now widely accepted by the physics community that the most recent high-precision cosmological data implies that the Universe is currently experiencing a state of accelerated expansion [1, 2]. Such observations are consistent with the presence of a non-zero, but surprisingly small, cosmological constant. It has thus become a pressing challenge for theoretical physicists to come up with an explanation for the presence and size of such a contribution to the Universe's energy content.

Lately, reviving ideas that had been developped some twenty years ago [3, 4, 5], a promising avenue for tackling this problem has surfaced in the context of braneworld scenarios [6, 7, 8], more precisely in codimension-two braneworlds [11, 12, 13, 14]. The key observation is that in such models, the four dimensional expansion rate is not directly related to the brane tension, but is rather a function of bulk parameters. This feature is intimately related to the fact that codimension-two objects induce a conical singularity, but do not otherwise curve the surrounding space.

While the cosmology of codimension-one branes have been well studied [9, 10], the same can not be said of codimension-two branes, for which research has so far centered almost exclusively on maximally symmetric solutions with pure tension branes. The reason for this,

as we will show, is that for more general types of brane energy content, the metric becomes singular at the position of the brane(s), so that some sort of regularisation scheme is needed in order to study the behaviour of these models in a more generalised setting.

We present in this talk summary of work done in [15], where we have constructed a thick braneworld model in order to answer the following questions:

- does the cosmology of codimension-two braneworlds follow the expected FRW behaviour?
- how does the deficit angle react to a sudden change in the brane tension, as happens e.g. during a phase transition?

2 Codimension-two branes

The feature which makes codimension-two branes an attractive possibility for addressing the cosmological constant problem is the fact that the brane does not curve the surrounding internal space, apart from inducing a conical defect proportional to the brane's tension. This leads to an automatic cancellation of the tension's contribution to the action [14], so that the four dimensional expansion rate is not obviously dependent on the vacuum energy of any field theory residing on the brane.

However, in contrast with codimension-one branes, it is not obvious that one can put matter with an equation of state different from pure tension on a codimension-two brane [14, 16]. The reason for this is that in order to do so, one has to drop the requirement that the metric be regular at the position of the brane. In order to circumvent this difficulty, Bostock *et.al.*[16] have suggested adding Gauss-Bonnet terms to the bulk gravitational action. We have chosen to take a different approach, accepting that the singularity will be present, as happens in any theory featuring point sources, but dealing with it by constructing a thick brane model whose zero thickness limit we will eventually want to study [15].

We will be working with a particular codimension-two braneworld where the bulk matter content consists of a cosmological constant and a two-form. The interplay between the two compactifies and stabilises the internal space [17], which is essentially spherical. However, the presence of branes at the poles will induce a deficit angle, so that the internal space will effectively look like an american football, or rugby ball. The relation between the brane tension, denoted by $\sigma^{(4)}$, and the deficit angle is given by

$$\Delta = 8\pi G_6 \sigma^{(4)}. \tag{1}$$

It can be shown that such a construction has solutions where the four dimensional space is flat, dS, or AdS, depending on bulk parameters. Indeed, with the two-form given by $F_{ab} = \beta\sqrt{|g_2|}\epsilon_{ab}$ where g_2 is the internal space metric, and ϵ_{ab} the two dimensional antisymmetric tensor, one finds the following relation between the four dimensional expansion rate, H, and bulk parameters

$$H^2 = \frac{4}{3}\pi G_6 \left(\Lambda_6 - \frac{\beta^2}{2}\right). \tag{2}$$

If one adds supersymmetry to the picture[18, 19], the dilaton equation of motion forces one to have

$$\Lambda_6 = \frac{\beta^2}{2} \tag{3}$$

so that the flat brane solutions are actually singled out. (We will not however be including supersymmetry in the construction we consider here).

3 Regularised branes

In order to study what happens in this model if we put more general types of matter on the brane, we will smooth out the singularity at the brane by constructing an equivalent thick brane model[15]. Furthermore, we will be treating matter as a perturbation to the static "football" shaped background described in the previous section. Therefore, our metric ansatz will be

$$\begin{aligned} ds^2 &= -e^{2(N_0(r)+N_1(r,t))}dt^2 + a_0^2(t)e^{2(A_0(r)+A_1(r,t))}d\vec{x}^2 + (1+B_1(r,t))^2 dr^2 \\ &\quad + e^{2(C_0(r)+C_1(r))}d\theta^2 + 2E_1(r,t)\,dr\,dt. \end{aligned} \tag{4}$$

The bulk action can be written as

$$S_{bulk} = \int d^6x\sqrt{-g}\left(\frac{R}{16\pi G_6} - \Lambda_6 - \frac{1}{4}F^{ab}F_{ab}\right) \tag{5}$$

We assume that the only nonvanishing component of the vector potential is $A_\theta(r,t) = A_\theta^{(0)}(r) + A_\theta^{(1)}(r,t)$. For the sake of generality, we include a possible perturbation of the 6D cosmological constant, $\Lambda_6 \to \Lambda_6 + \delta\Lambda_6$. The full stress-energy tensor is taken to be of the form

$$T^a_b(r,t) = t^a_b(r,t) + \theta(r_0(t)-r)S^a_b(r,t) + \theta(r - r_*(t))S_{*}{}^a_b(r,t) \tag{6}$$

where t^a_b refers to the bulk content, while S^a_b is the core stress energy, given by

$$\begin{aligned} S^t_t &= -\sigma - \rho; \qquad S^x_x = -\sigma + p; \qquad S^r_r = 0 + p^r_r; \\ S^\theta_\theta &= 0 + p^\theta_\theta; \qquad S^r_t = 0 + p^r_t; \qquad S^t_r = 0 + p^t_r; \\ S_{*}{}^t_t &= -\sigma - \rho_*; \qquad S_{*}{}^x_x = -\sigma + p_* \qquad S_{*}{}^r_r = 0 + p_{*}{}^r_r; \\ S_{*}{}^\theta_\theta &= 0 + p_{*}{}^\theta_\theta; \qquad S_{*}{}^r_t = 0 + p_{*}{}^r_t; \qquad S_{*}{}^t_r = 0 + p_{*}{}^t_r. \end{aligned} \tag{7}$$

and we treat the time dependence of the thickness as a perturbation, so that $r_0(t) = r_0 + \Delta r_0(t)$, $r_*(t) = r_* + \Delta r_*(t)$ and

$$\theta(r_0(t) - r) = \theta(r_0 - r) + \delta(r_0 - r)\Delta r_0(t) + O(\Delta r_0^2) \tag{8}$$

$$\theta(r - r_*(t)) = \theta(r - r_*) - \delta(r - r_*)\Delta r_*(t) + O(\Delta r_*^2). \tag{9}$$

so that effectively, we can write the stress-energy tensor as

$$t^a_b + s^a_b + s_{*}{}^a_b \tag{10}$$

with, e.g.,

$$\begin{aligned} s^t_t &= -\sigma\theta(r_0 - r) + [-\rho\theta(r_0 - r) - \sigma\delta(r_0 - r)\Delta r_0(t)] & (11) \\ s^i_i &= -\sigma\theta(r_0 - r) + [p\theta(r_0 - r) - \sigma\delta(r_0 - r)\Delta r_0(t)] & (12) \end{aligned}$$

and similarly for all other terms.

Here σ represents the tension of the regularised brane, and ρ, p represent contributions from ordinary matter on the standard-model (SM) brane, while starred quantities refer to matter on a hidden brane which is antipodal to the SM brane on the two-sphere bulk.

The subscripts on the metric and gauge field perturbations indicate their order in a perturbative series in powers of ρ. We will furthermore assume that time derivatives of the perturbations are of $O(\rho^{3/2})$, which is implied by the usual law for conservation of energy $\dot{\rho} \sim (\dot{a}/a)\rho \sim \rho^{3/2}$.

4 Generalised cosmology

Solving the system we have just described to linear order in the perturbations, one finds that the Friedmann equations emerge through the imposition of boundary conditions. (See [15] for details). One further has to be careful to express all results in terms of effective four dimensional quantities that are relevant to observers confined to the brane rather than the six dimensional quantities that were defined above. This is done by integrating the 6D quantities over the thickness of the brane,

$$S^{(4)}{}^a_b = 2\pi \int_0^{r_0} dr\, \sqrt{|g_2|}\, S^{(6)}{}^a_b \tag{13}$$

which perturbatively leads to

$$\begin{aligned} \sigma^{(4)} + \rho^{(4)} &= 2\pi \int_0^{r_0} dr\, e^{C_0}(1 + B_1 + C_1)(-s^t_t) & (14) \\ -\sigma^{(4)} + p^{(4)} &= 2\pi \int_0^{r_0} dr\, e^{C_0}(1 + B_1 + C_1)(s^i_i) & (15) \end{aligned}$$

and similarly for the other brane.

Also, the 4D Newton constant is related to the 6D one by dimensional reduction,

$$\begin{aligned} G_6 &= G_4 \times V & (16) \\ &= G_4 \int_0^{2\pi} d\theta \int_0^{\pi/k - \phi} e^{C_0(r)} & (17) \\ &= G_4 \times \frac{4\pi}{k^2 \bar{k}^2}(\bar{k}^2 + (k^2 - \bar{k}^2)\cos(kr_0)) & (18) \end{aligned}$$

where we neglect corrections of $O(\rho)$.

Expressing the Friedmann equations in terms of these effective four dimensional quantities leads to

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G_4}{3}\left(\rho^{(4)} + \rho_*^{(4)} + \Lambda_{\rm eff}\right) \tag{19}$$

$$\frac{\ddot{a}_0}{a_0} = \left(\frac{\dot{a}_0}{a_0}\right)^2 - 4\pi G_4\left(\rho^{(4)} + p^{(4)} + \rho_*^{(4)} + p_*^{(4)}\right) \tag{20}$$

$$\dot{\rho}^{(4)} = -3\frac{\dot{a}_0}{a_0}\left(\rho^{(4)} + p^{(4)}\right) \tag{21}$$

$$\dot{\rho}_*^{(4)} = -3\frac{\dot{a}_0}{a_0}\left(\rho_*^{(4)} + p_*^{(4)}\right). \tag{22}$$

The appearance of the constant Λ_{eff} simply reflects the fact that our choice to expand around a static background solution was arbitrary, and we could just as well have expanded around one of the (A)dS solutions instead. The important point is that these are simply the standard Friedmann equations, with the possible added contribution from matter on a hidden brane, which shows that we do indeed recover standard cosmology from codimension-two braneworlds.

5 Generalised deficit angle

We must now consider how the deficit angle should be defined in the case we are considering, where the brane is thick. From the bulk point of view, the radial distance from the brane at $r = R$ is $R - r_0$. The circumference of a circle of radius R is $2\pi g_{\theta\theta}(R,t)$, while the circumference of the brane is $2\pi g_{\theta\theta}(r_0,t)$. If there is no matter on the brane, so that the internal space is perfectly spherical, we would expect that as $r_0 \to 0$ and $R \to 0$,

$$2\pi[g_{\theta\theta}(R,t) - g_{\theta\theta}(r_0,t)] = 2\pi(R - r_0). \tag{23}$$

On the other hand, if there is matter on the brane, it will modify the previous relation to read

$$2\pi(g_{\theta\theta}(R,t) - g_{\theta\theta}(r_0,t)) = 2\pi(R - r_0)\left(1 - \frac{\Delta}{2\pi}\right). \tag{24}$$

Thus we can define the deficit angle as

$$\Delta \equiv 2\pi \lim_{R\to 0}\left[\lim_{r_0\to 0} 1 - \frac{g_{\theta\theta}(R,t) - g_{\theta\theta}(r_0,t)}{R - r_0}\right]. \tag{25}$$

Plugging in the results one gets from solving the linearised equations of motion[15], we find the following generalised expression for the deficit angle

$$\Delta = 2\pi G_6\left(4\sigma^{(4)} + \rho^{(4)} - 3p^{(4)}\right) \tag{26}$$

and similarly around the other brane.

6 Discussion

The first point we wish to emphasise is that the apparent obstruction to having arbitrary types of matter on a codimension-two brane is seen to stem from the unreasonable expectation that the metric should be regular at the position of the brane. Once this assumption is dropped, there is no such obstruction, and it is furthermore possible to smooth out the singularities in the metric by considering thick branes.

Our results show that the answers to our original questions are

- codimension-two braneworlds *do* lead to FRW cosmology;
- the deficit angle will respond dynamically to a change in the brane tension.

Unfortunately, our results also show that there can not be a self-tuning mechanism leading to a solution to the cosmological constant problem. This might seem surprising given the fact that the cancellation between the deficit angle and brane stress-energy which initially led to this hope still holds in the generalised model. However, closer inspection shows that it is the additional tuning between the gauge field strength and bulk cosmological constant which is spoiled by matter perturbations and leads to expansion on the brane. (See [15] for a more thourough discussion).

We thus confirm recent work on the subject[20, 21, 22] which also pointed to the conclusion that in the context of Einstein gravity, codimension-two braneworlds did not lead to self-tuning, as can be seen from the absence of massless scalars in the low energy effective theory.

While this conclusion definitely rules out codimension-two braneworlds in Einstein gravity as solutions to the cosmological constant problem, the possibility remains open that similar supersymmetric models[18, 19] might prove more successful [1].

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